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Critical conditions for bed material movement in steep, boulder-bed streams

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ABSTRACT In bed materials with nonuniform size distributions, particles smaller than a particular reference size are relatively difficult to move while particles larger are relatively easy to move. In addition, at steep slopes, the critical conditions for movement are best predicted by an approach based on water discharge rather than the Shields shear stress. These two conditions are combined empirically to form a method of calculating critical unit water discharge for particle movement in steep channels with coarse nonuniform bed material. The relationship between particle size and critical unit water discharge is evaluated using field data and is found to vary with bed material size distribution. The reference diameter is of the order of bed material D_{50} size. An independent test of the relationships indicates good results if the unit water discharge is based on the active width for bed load transport rather than on flow width.

INTRODUCTION

The standard means of calculating the critical flow conditions for initiation of bed material movement in a channel is the Shields (1936) equation:

$$\frac{\tau_c}{(\rho_s - \rho)\ g\ D} = \tau_{*c} \qquad (1)$$

where τ_c = critical shear stress; ρ_s = bed material density; ρ = water density; g = acceleration due to gravity; D = bed material particle diameter; and τ_{*c} = the Shields parameter. For gravels and coarser materials with uniform size distributions, the parameter is typically assigned a constant value in the range 0.04-0.06. For bed materials with nonuniform size distributions, though, the parameter varies as interaction between the different particle sizes affects the critical conditions for each size fraction. Recent research has also indicated that methods based on shear stress may not be suitable for steep, boulder-bed channels. This paper therefore examines and

develops an alternative approach based on flow discharge, which may be more appropriate for the steep channels with coarse, nonuniform bed materials characteristic of mountain regions. The approach is developed using field data and is subjected to an independent test.

FACTORS AFFECTING THE CRITICAL CONDITIONS

Nonuniform size distribution

For a nonuniform bed material the various particle sizes are brought into motion over a range of discharges, rather than at a single flow condition. If the individual size fractions had no influence on each other, the force required to initiate movement of a given size would be equal to that required to move the same size in a bed composed of uniform material of that size. The Shields parameter would then take the same standard value for each size fraction. Several studies have shown, though, that the stability of a particle is affected by the position of its size within the overall size distribution, relative to a reference size (Egiazaroff, 1965; White & Day, 1982). Particles smaller than this size tend to be sheltered behind the larger particles and require a stronger flow (with larger Shields parameter) to set them in motion than would be necessary for uniform materials of the same size. Conversely, particles larger than this size tend to project into the flow and can be moved by flows weaker (with smaller Shields parameter) than would be necessary for uniform materials of the same size (e.g. Fenton & Abbott, 1977; White & Day, 1982; Proffitt & Sutherland, 1983). Particles of the reference size are unaffected by the hiding/exposure effect and behave as if in a bed of uniform material. Empirically, the reference size is of the order of D_{50}, that size of particle median axis for which 50% of the particles are finer (e.g. Çeçen & Bayazit, 1973; Proffitt & Sutherland, 1983).

Andrews (1983) has quantified the effect on critical shear stress using field data. For the range $0.3<D_i/D_{50}<4.2$, he found that

$$\tau_{*ci} = 0.0834\ (D_i/D_{50})^{-0.872} \qquad (2)$$

where τ_{*ci} = the average critical Shields parameter for particles of size D_i in the surface or armour layer of the bed; i = the size fraction; and D_{50} refers to the subsurface or parent bed material.

High slope and large-scale roughness

Several studies have shown that the critical Shields parameter rises to values of 0.1 and higher as slope increases above about 1% and the ratio of depth to particle size falls below about 10 (Ashida & Bayazit, 1973; Bathurst et al., in press). Bathurst et al. (in press) therefore adopted the Schoklitsch (1962) approach to prediction of the critical flow conditions, based on unit water discharge rather

than shear stress. Using flume data for bed materials with relatively uniform size distributions they developed the empirical relationship

$$q_c = 0.15\ g^{0.5}\ D^{1.5}\ S^{-1.12} \qquad (3)$$

where q_c = critical water discharge per unit width; and S = slope. The equation was derived for the range of slopes $0.25 \leq S \leq 20\%$ and particle sizes $3 \leq D \leq 44$ mm and for ratios of depth to particle size as low as 1.

GENERAL APPROACH

The aim of this study is to combine the two strands of research just described into a method of predicting the critical unit water discharge for each particle size in steep, boulder-bed streams. Adopting a nondimensional form similar to that of equation (2), the proposed relationship is

$$q_{ci} = q_{cr}\ (D_i/D_r)^b \qquad (4)$$

where q_{ci} = the critical unit discharge for movement of particles of size D_i; q_{cr} = the critical unit discharge for the reference particle size D_r which is unaffected by the hiding/exposure effect; and b = an exponent. An independent evaluation of q_{cr} is given by equation (3). Owing to the complexity of the particle entrainment process, quantification of equation (4) is carried out empirically using field data.

DATA COLLECTION

The necessary measurements of bed load particle sizes, water discharges and channel characteristics were carried out by teams from the British Institute of Hydrology and Colorado State University in a study of the Roaring River, a boulder-bed stream with a snowmelt regime in the Rocky Mountain National Park, Colorado, USA (Bathurst et al., 1986a; Pitlick & Thorne, in press).

The particle size characteristics were obtained from samples of the bed load collected using a Helley-Smith sampler with an aperture of 150 mm. Each sample was composed of two or three subsamples from points across the stream. Typically the time taken for the total sample was 2 or 3 minutes but could be as little as 30 seconds at high bed load transport rates.

Flow discharge, accurate to about 10%, was obtained from stage measurements converted via a stage-discharge relationship. Channel data (width and slope) were obtained by normal surveying techniques. Bed material size distributions for the bed surface were obtained using Wolman's (1954) grid sampling method, for samples of 100 particles. Further information on the field techniques is given in Bathurst et al. (1986b).

Measurements were made at two sites, the Ypsilon Lake Trail bridge and the Fall River Road bridge, during the snowmelt seasons of 1984 and 1985. The sites are about 1 km apart, just upstream of the confluence of the Roaring River with the Fall River. Site data are given in Table 1. Those for the Ypsilon Lake Trail bridge pertain to a 30-m reach of channel immediately upstream of the bridge. Those for the Fall River Road bridge apply to a 40-m shoal reach about 50 m upstream of the sampling point on the bridge. The shoal was thought to regulate the movement of bed material past the bridge in a closer correspondence with flow discharge than did the more armoured length of channel immediately upstream of the bridge.

DATA PREPARATION

Necessary data for quantifying equation (4) are pairs of sizes D_i of the particle median axes and the critical unit water discharges q_{ci} associated with movement of those sizes. For two reasons, though, these pairs could not be formed simply from the maximum particle size and the water discharge recorded for each bed load sample.

(a) Because the Helley-Smith sampler did not catch all the bed load passing a section and because the samples represented a restricted time period, the maximum particle size in a given sample was not necessarily equal to the maximum size that could be set in motion by the recorded water discharge. In order to obtain a more representative value, therefore, the maximum particle size was taken, not from each individual bed load sample, but from groups of samples collected at intervals of about 30 minutes during periods of continuous sampling, typically 2 to 12 hours in length.

TABLE 1 Channel parameters at the sampling sites

Period of validity	Channel width (m)	Channel slope (m m^{-1})	Bed material size (mm)		
			D_{16}	D_{50}	D_{84}
Ypsilon Lake Trail bridge					
1984	6.25	0.0383	48	106	208
18/5- 6/6/85	6.25	0.0360	52	104	211
7/6-12/6/85	6.25	0.0339	48	109	223
Fall River Road bridge					
1984	10.9	0.0460	25	72	217
18/5-27/5/85	6.1	0.0523	32	77	156
27/5- 6/6/85	6.1	0.0470	-	-	-
7/6-12/6/85	6.1	-	58	140	251

Average slope used in calculations for the Ypsilon Lake Trail bridge site is 0.036 m m^{-1}.

(b) On the falling limb of a hydrograph, even if a given sample did include the maximum particle size in motion, that particle might have been set in motion by an earlier, higher flow than that recorded at the time of sampling. Consequently samples for such flows were considered only if the maximum particle size was unusually large, generally exceeding the D_{50} size of the bed material. These particles were assumed to have been set in motion by the peak discharge of the hydrograph and the selected maximum size was therefore paired with that discharge. For all other periods of flow, the critical discharge was taken to be that recorded at the time of sampling for the selected particle size. In each case, the unit water discharge was obtained by dividing the total discharge by channel width. Change of width with discharge at a site was relatively small and was therefore neglected.

DATA ANALYSIS

The relationships between D_i and q_{ci} for each site are plotted in Figures 1 and 2. At the Ypsilon Lake Trail bridge site there was little variation in bed material and channel characteristics over the entire fieldwork period and all the data for that site can therefore be analyzed together. At the Fall River Road bridge site the bed material and channel characteristics (and thence the critical conditions for bed material movement) varied as a result of the construction of a footbridge across the shoal during the 1984-1985 winter and the subsequent reforming of the bed during the

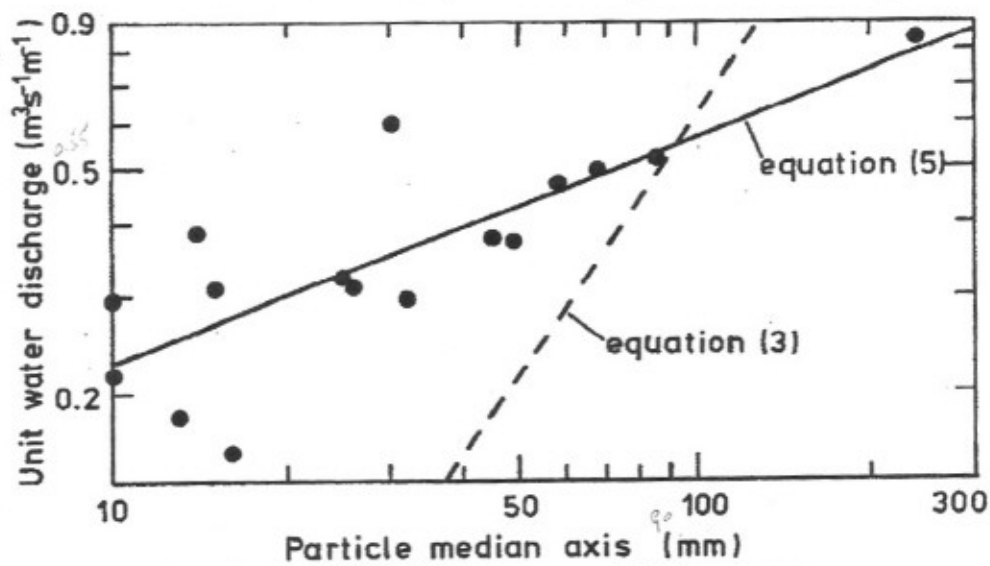

FIG.1 Relationship between critical unit water discharge and size of particle median axis for the Ypsilon Lake Trail bridge site. Equation (5) is fitted to the data and compared with equation (3).

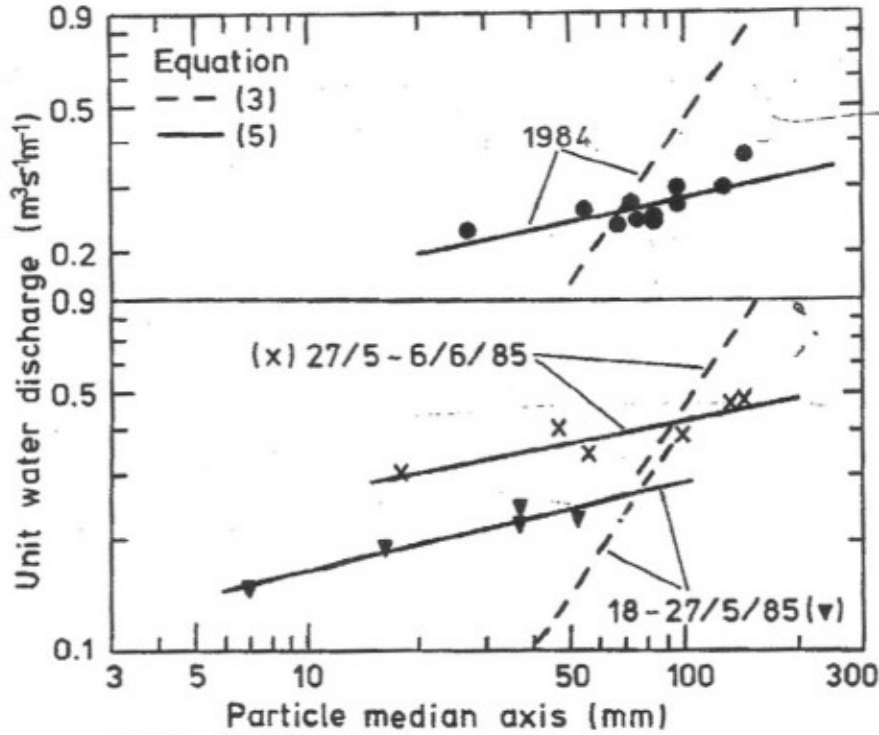

FIG.2 Relationship between critical unit water discharge and size of particle median axis for the Fall River Road bridge site. Equation (5) is fitted to the data and compared with equation (3). Symbols indicate period of validity.

major snowmelt floods of 27 May and 6-8 June 1985. The data for this site are therefore analyzed separately for 1984 and the periods 18-27 May and 27 May - 6 June 1985.

Generally, q_{ci} and D_i increase together. Simplified versions of equation (4), of the form

$$q_{ci} = a\, D_i^b \qquad (5)$$

were therefore fitted to the data for each case using regression analysis. Details are given in Table 2.

Also plotted in Figures 1 and 2 are the lines of equation (3) showing the relationships between q_{ci} and D_i for uniform bed material at the channel slopes appropriate to each site and period. Comparison of the plotted equations (3) and (5) neatly illustrates the hiding/exposure effect. Particles smaller than a reference size require larger discharges to initiate their motion if they are in a nonuniform rather than a uniform bed material. For particles larger than the reference size, the converse is true. The

TABLE 2 Parameters of the equation $q_{ci} = aD_i^b$ fitted for each sampling site and the corresponding reference particle size

Period of validity	Equation parameters a	Equation parameters b	Equation r^2 (%)	Reference particle size (mm)
Ypsilon Lake Trail bridge				
1984 & 1985	0.0933	0.392	61.1	93 - 0.84
Fall River Road bridge				
1984	0.103	0.220	49.9	68
18/5-27/5/85	0.0955	0.239	91.0	76
27/5- 6/6/85	0.168	0.201	75.5	94

q_{ci} has dimensions $m^3s^{-1}m^{-1}$; D_i has dimensions mm.

reference size itself is defined by the intersection of the two lines. The values are given in Table 2 and confirm the findings of earlier studies that the reference size is of the order of the bed material D_{50}.

A possible relationship between the exponent b in equations (4) and (5) and the ratio D_{84}/D_{16} is explored by adding the constraint, from equation (3), that $b = 1.5$ when $D_{84}/D_{16} = 1$, i.e. for uniform material. The resulting approximate expression, obtained empirically from Figure 3, is

$$b = 1.5(D_{84}/D_{16})^{-1} \qquad (6)$$

(Pending the availability of further data, the exponent is given the value -1 for convenience.) Conceptually, equation (6), allied with equation (4), indicates that, the wider the size distribution, the stronger is the hiding/exposure effect and the narrower is the range of discharges over which all particle sizes are brought into motion.

From the above, the critical flow conditions for each particle size can be calculated from equation (4), with the exponent b given by equation (6) and q_{cr} calculated by equation (3), assuming that $D_r = D_{50}$ for the surface layer of the bed material.

TEST OF DERIVED RELATIONSHIPS

Data presented by Bulgurlu (1977) for the River Gaula in Norway are used in an independent test of the derived relationships. Bankfull width at the sampling section was 150 m, average slope was about 0.0024 m m^{-1} and the bed material D_{16}, D_{50} and D_{84} values, taken from the plotted size distribution curve, are 27, 75 and 140 mm respectively. Some 52 bed load samples were collected using Arnhem and Mühlhofer samplers for flow discharges in the range

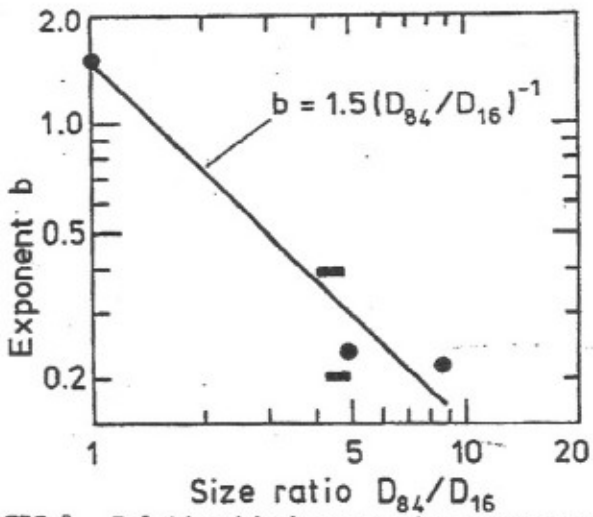

FIG.3 Relationship between the exponent b of equation (5) and the bed material size ratio D_{84}/D_{16}. Bars indicate where the size ratio is given as a range.

150-1050 m^3s^{-1}.

Particle size distributions for the bed load are presented by Bulgurlu as ensembles of all the samples analyzed for three subdivisions of the discharge range. It is assumed here that the maximum discharge in each range is the critical discharge for movement of the maximum particle size in each ensemble, giving the pairs: discharge = 450 m^3s^{-1}, D = 43 mm; discharge = 700 m^3s^{-1}, D = 115 mm; discharge = 1050 m^3s^{-1}, D = 170 mm.

Each of these discharges was converted into a unit discharge by dividing by the flow surface width. For bed load transport calculations, though, the unit discharge should probably be determined for that part of the flow above the active width of the bed, over which there is bed load movement (e.g. Schoklitsch, 1962). For the Roaring River there was little difference between the active and surface widths. For the Gaula the active width was about 60% of the surface width, although approximately 91 to 99% of the total discharge still passed over the active width. A second set of unit discharges was therefore obtained by dividing the total discharges by the active width of bed. The total discharges were not reduced to allow for the discharge passing over the inactive bed and the second set of unit discharge values are therefore slightly too high.

Calculated values of critical unit discharge were obtained using equations (3), (4) and (6) and are compared with the two sets of measured values in Figure 4. Agreement is close where the measured unit discharge is based on the active width but is poor where the flow surface width is used. This supports the use of the active width in determining the unit discharge.

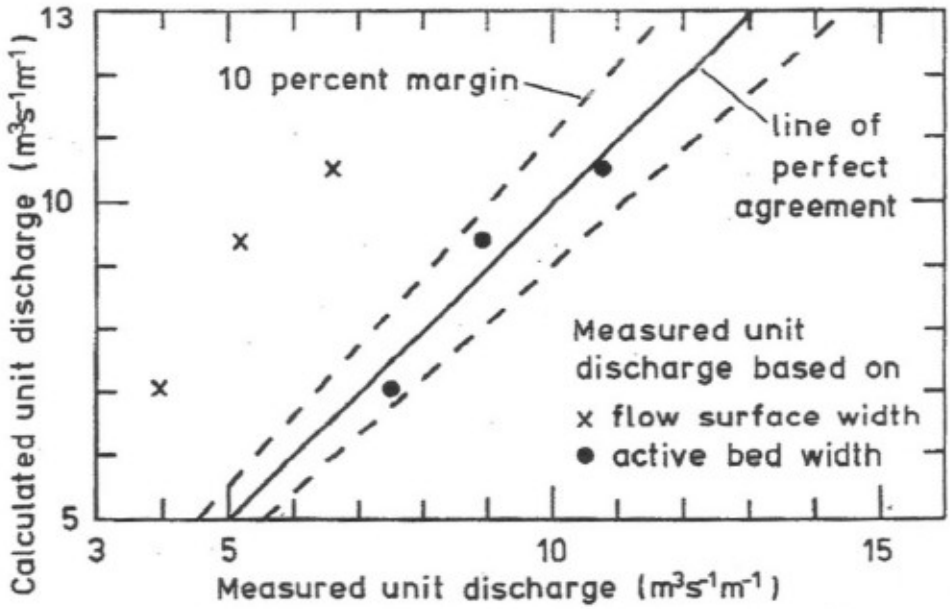

FIG. 4 Comparison of calculated and measured values of critical unit water discharge for the River Gaula.

CONCLUSION

Equations (3), (4) and (6) are presented as a means of calculating the critical conditions for particle movement in steep channels with coarse, nonuniform bed material. As such they should be of use in sediment transport models and general hydraulic calculations. The test with the Gaula data provides encouraging support for the relationships but indicates that the role of the active bed width needs more detailed investigation. Also, the relationships are derived from rather limited data and should clearly be verified and refined in further field tests.

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REFERENCES

Andrews, E.D. (1983) Entrainment of gravel from naturally sorted riverbed material. Geol. Soc. Am. Bull. 94, 1225-1231.

Ashida, K. & Bayazit, M. (1973) Initiation of motion and roughness of flows in steep channels. Proc. 15th Congr. IAHR, Istanbul, vol. 1, 475-484.

Bathurst, J.C., Leeks, G.J.L. & Newson, M.D. (1986a) Relationship between sediment supply and sediment transport for the Roaring River, Colorado, USA. In : Drainage Basin Sediment Delivery (Proc. Albuquerque Symp., Aug. 1986), 105-117. IAHS Publ. no. 159.

Bathurst, J.C., Leeks, G.J.L. & Newson, M.D. (1986b) Field measurements for hydraulic and geomorphological studies of sediment transport - the special problems of mountain streams. In : Measuring Techniques in Hydraulic Research (Proc. IAHR Delft Symp., April 1985), 137-151. A.A. Balkema, Rotterdam, The Netherlands.

Bathurst, J.C., Graf, W.H. & Cao, H.H. (In press) Bed load discharge equations for steep mountain rivers. In : Sediment Transport in Gravel-bed Rivers (ed. by C.R. Thorne, J.C. Bathurst & R.D. Hey). John Wiley & Sons, Chichester, UK.

Bulgurlu, B. (1977) A study of sediment transport in River Gaula. PhD thesis. Univ. Trondheim, Norway.

Çeçen, K. & Bayazit, M. (1973) Critical shear stress of armored beds. Proc. 15th Congr. IAHR, Istanbul, vol. 1, 493-500.

Egiazaroff, I.V. (1965) Calculation of nonuniform sediment concentrations. J. Hydraul. Div. ASCE 91(HY4), 225-247.

Fenton, J.D. & Abbott, J.E. (1977) Initial movement of grains on a stream bed : the effect of relative protrusion. Proc. Roy. Soc. Lond. A352, 523-537.

Pitlick, J.C. & Thorne, C.R. (In press) Sediment supply, movement and storage in an unstable gravel-bed river. In : Sediment Transport in Gravel-bed Rivers (ed. by C.R. Thorne, J.C. Bathurst & R.D. Hey). John Wiley & Sons, Chichester, UK.

Proffitt, G.T. & Sutherland, A.J. (1983) Transport of non-uniform sediments. J. Hydraul. Res. 21(1), 33-43.

Schoklitsch, A. (1962) Handbuch des Wasserbaues (Handbook of Water Engineering), 3rd edn. Springer-Verlag, Vienna.

Shields, A. (1936) Anwendung der Ähnlichkeitsmechanik und Turbulenzforschung auf die Geschiebebewegung (Application of similarity mechanics and turbulence research to bed load movement). Rep. 26, Mitteil. Preuss. Versuchsanst. Wasserbau und Schiffsbau, Berlin.

White, W.R. & Day, T.J. (1982) Transport of graded gravel bed material. In : Gravel-bed Rivers (ed. by R.D. Hey, J.C. Bathurst & C.R. Thorne), 181-213. John Wiley & Sons, Chichester, UK.

Wolman, M.G. (1954) A method of sampling coarse river-bed material. Trans. AGU 35(6), 951-956.